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Multiple equilibria in Lucas (1990)'s optimal capital taxation model with endogenous learning^{*}

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Abstract

In the paper we solve the general case of the Lucas (1990) optimal capital taxation model with endogenous growth driven by endogenous learning. We prove Lucas (1990)'s conjecture on zero limiting capital tax and display the possibility of multiple equilibria (i.e., multiple BGPs) in the model.

Keywords: Multiple Equilibria; Capital Income Tax; Endogenous Growth; Endogenous Learning.

JEL Classification Numbers: H21, E62.

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1 Introduction

Lucas (1990) contributes to examine the Chamley (1986)-Judd (1985) zero capital tax theorem in a model with endogenous growth driven by human capital accumulation. In his well-known paper, he presents the result of zero limiting capital tax for the case with exogenous human capital accumulation and then focuses on quantifying the welfare cost of capital taxation for the U.S. economy. For the more general case with endogenous human capital accumulation, however, he just conjectures that the zero limiting capital tax result still holds, without working out the details of the Ramsey problem. Furthermore, the balanced growth path is assumed (rather than proved) to be existent. It is useful and important to solve the Lucas (1990) model completely, just like what Lucas had said in his paper: "It would be a useful but difficult task to provide a full characterization of solutions to this maximum problem".

In the paper we want to solve the general case of the Lucas (1990) model, verify his conjecture on the result of zero limiting capital tax and prove the existence of the balanced growth path. Acturally, it is not easy to solve explicitly the general case because an integral equation (rather than a differential or algebraic equation) should be incorporated as another constraint in the Ramsey problem and the Lagrangian/Hamiltonian cannot deal with it directly. To overcome this difficulty, by following the literature on endogenous time preference, such as Uzawa (1968), we define a new state variable and transform the integral equation into a differential equation and an algebraic equation. Hence we can solve the model explicitly and prove Lucas (1990)'s conjecture. When proving the existence of the balanced growth path, we utilize the particular functional forms of the production function, learning technology and the utility function used in the Lucas (1990) model, and find out that there may exist multiple equilibria (i.e., multiple balanced growth paths) with zero capital tax.

The rest of the paper is organized as follows. Section 2 presents the Lucas (1990) model with endogenous learning. We solve the general case of the Ramsey problem and prove Lucas (1990)'s conjecture in section 3. In section 4 we examine the existence of the balanced growth path and refer to the possibility of multiple equilibria. Section 5 concludes.

2 The Lucas (1990) model with endogenous learning

In this section, we review briefly Lucas (1990)'s optimal capital taxation model with endogenous growth and present the dynamic system of Euler equations describing the market equilibrium of the model economy.

Households. The representative household maximizes the objective function

$$\int_{t=0}^{\infty} e^{-(\rho-\lambda)t} U(c(t), x(t)) dt, \quad (1)$$

subject to the flow budget constraint

$$\dot{k}(t) = r(t)k(t) + w(t)u(t)h(t) + b(t) - c(t) - \lambda k(t), \quad (2)$$

and human capital accumulation equation

$$\dot{h}(t) = h(t)G(v(t)), \quad (3)$$

where $\rho(>0)$ is time discount rate, λ is population growth rate; $c(t)$ is per capita consumption, $k(t)$ and $h(t)$ are per capita stocks of physical and human capital, $b(t)$ is per capita transfer payments from the government; $x(t) = (1 - u(t) - v(t))$, $u(t)$ and $v(t)$ are time for leisure, work and learning, with the total time endowment 1 at each t ; $r(t)$ and $w(t)$ are the interest rate and real wage net of taxes; the initial values of stocks of physical and human capital $k(0)$ and $h(0)$ are exogenously given. The current period utility function has the constant elasticity form, i.e., $U(c, x) = (c\varphi(x))^{1-\sigma} / (1-\sigma)$, and the learning/human capital accumulation technology has the form of $G(v) = Dv^\gamma$, with positive technology parameters D and γ .

Application of the Pontryagin's maximum principle of optimal control leads to the first order necessary conditions:

$$e^{-(\rho-\lambda)t} \frac{U_c(c(t), x(t))}{U_c(c(0), x(0))} = \exp \left\{ - \int_{s=0}^t (r(s) - \lambda) ds \right\}, \quad (4)$$

$$\frac{U_x(c(t), x(t))}{U_c(c(t), x(t))} = w(t) h(t), \quad (5)$$

$$w(t) h(t) = G'(v(t)) \int_{s=t}^{\infty} \exp \left\{ - \int_{v=t}^s (r(v) - \lambda) dv \right\} w(s) u(s) h(s) ds. \quad (6)$$

Equation (4) tells that the marginal rate of substitution between consumption at time 0 and t must equal the relative prices of these two goods. Equation (5) shows that the marginal rate substitution between leisure and consumption must be equal to the real wage. Equation (6) is an integral equation displaying no arbitrage for optimal income-directed time allocation in producing final goods or accumulating human capital.

Firms. The representative firm employs capital $k(t)$ and effective labor $u(t) h(t)$, produces the final good with the linearly homogenous production technology $F(k(t), u(t) h(t)) = k(t)^\beta [u(t) h(t)]^{1-\beta}$, $\beta \in (0, 1)$, and maximizes its profit. Perfect competition ensures that both factors are paid their marginal products. Hence,

$$w(t) = (1 - \theta) F_n(k(t), u(t) h(t)), r(t) = (1 - \tau) F_k(k(t), u(t) h(t)), \quad (7)$$

where θ is the tax rate on labor income and τ is the tax rate on capital income.

Government. The government levies flat-rate taxes on labor and capital incomes with full commitment to finance its consumption, $g(t)$, and transfer payments, $b(t)$, and runs a balanced budget constraint,

$$g(t) + b(t) = \frac{\theta}{1 - \theta} r(t) k(t) + \frac{\tau}{1 - \tau} w(t) u(t) h(t). \quad (8)$$

Combining equations (2) and (8) and plugging (7) in it, we recover the social resource constraint

$$c(t) + k(t) + \lambda k(t) + g(t) = F(k(t), u(t) h(t)). \quad (9)$$

Then, the marginal conditions (4)-(7), together with the equations of motion (3) and (9) for the two kinds of capital, form a system of Euler equations that describes the competitive equilibrium of this model economy given the initial stocks of physical and human capital.¹

3 The Ramsey problem and zero limiting capital tax

In this section we formulate the Ramsey problem by utilizing the Primal approach developed by Atkinson and Stiglitz (1980) and used by Lucas and Stokey (1983) and solve it by defining a new state variable and changing the integral equation into a differential equation and an identity.

Firstly, we integrate the flow budget constraint of the household and derive the present-value budget constraint

$$\int_{t=0}^{\infty} \exp \left\{ - \int_{s=0}^t (r(s) - \lambda) ds \right\} [c(t) - w(t) u(t) h(t) - b(t)] ds = k(0). \quad (10)$$

Substituting (4) and (5) into (10) gives rise to the implementability condition

$$\int_{t=0}^{\infty} e^{-(\rho-\lambda)t} \{U_c(t) [c(t) - b(t)] - U_x(t) u(t)\} ds = U_c(0) k(0). \quad (11)$$

Putting (4) and (5) into (6), we obtain the following key integral equation in the more general model with endogenous learning

$$U_x(c(t), x(t)) = G'(v(t)) \int_{s=t}^{\infty} e^{-(\rho-\lambda)(s-t)} u(s) U_x(c(s), x(s)) ds, \quad (12)$$

which should be incorporated in solving the Ramsey problem. Therefore, the Ramsey problem is: maximize (1) subject to (3), (9), (11) and (12). Notice that equation (12) is an integral equation which is hard to be incorporated into a Lagrangian. To overcome the difficulty, we define a new state variable

$$m(t) \equiv \frac{U_x(c(t), x(t))}{G'(v(t))}. \quad (13)$$

¹ If setting the tax rates τ and θ equal to zero, these same equations also serve to characterize the first-best allocation.

Substituting it back into equation (12), we derive a differential equation about $m(t)$, namely,

$$\dot{m}(t) = [\rho - \lambda - u(t)G'(v(t))]m(t). \quad (14)$$

In solving the Ramsey problem, we can replace the integral equation (12) with equations (13) and (14).

Then we construct the Lagrangian for the government's maximum problem as follows:

$$\mathcal{L} = \left\{ \begin{aligned} & e^{-(\rho-\lambda)t} W(c(t), 1-u(t)-v(t), \Phi) + \tilde{\lambda}(t)[\rho - \lambda - u(t)G'(v(t))]m(t) \\ & + \tilde{\mu}(t)[m(t)G'(v(t)) - U_x(c(t), x(t))] + \\ & \tilde{\eta}(t)[F(k(t), u(t)h(t)) - c(t) - \lambda k(t) - g(t)] + \tilde{\pi}(t)G(v(t))h(t) \end{aligned} \right\},$$

where

$$W(c(t), x(t), \Phi) \equiv U(c(t), x(t)) + \Phi[U_c(t)(c(t) - b(t)) - U_x(t)u(t)],$$

and Φ is a nonnegative multiplier, constant over time, and strictly positive if it is necessary to use any distorting taxes. Now we have three control variables $c(t), u(t), v(t)$, three state variables $k(t), h(t), m(t)$, three Hamilton multipliers $\tilde{\eta}(t), \tilde{\pi}(t), \tilde{\lambda}(t)$ and one Lagrange multiplier $\tilde{\mu}(t)$. By utilizing again Pontryagin's maximum principle of optimal control, we derive the following first order necessary conditions:

$$c(t) : \quad e^{-(\rho-\lambda)t} W_c(t) = \tilde{\mu}(t)U_{cx}(t) + \tilde{\eta}(t), \quad (15)$$

$$u(t) : \quad e^{-(\rho-\lambda)t} W_x(t) = -\tilde{\lambda}(t)G'(v(t))m(t) + \tilde{\mu}(t)U_{xx}(t) + \tilde{\eta}(t)F_n(t)h(t), \quad (16)$$

$$v(t) : \quad e^{-(\rho-\lambda)t} W_v(t) = -\tilde{\lambda}(t)u(t)G''(v(t))m(t) + \tilde{\mu}(t)[m(t)G''(v(t)) + U_{xx}(t)] + \tilde{\pi}(t)G'(v(t))h(t), \quad (17)$$

$$m(t) : \quad \tilde{\lambda}(t)[\rho - \lambda - u(t)G'(v(t))] + \tilde{\mu}(t)G'(v(t)) = -\dot{\tilde{\lambda}}(t), \quad (18)$$

$$k(t) : \quad \tilde{\eta}(t)[F_k(t) - \lambda] = -\dot{\tilde{\eta}}(t), \quad (19)$$

$$h(t) : \quad \tilde{\eta}(t)F_n(t)u(t) + \tilde{\pi}(t)G(v(t)) = -\dot{\tilde{\pi}}(t). \quad (20)$$

To find the limiting capital tax, we should derive the balanced growth path (henceforth, BGP). Suppose that there exists a BGP. We want to find a BGP satisfying: (1) $\dot{h}/h = \dot{c}/c = \dot{k}/k = \dot{g}/g = \dot{b}/b = \phi$; (2), $\mu(t), v(t)$ and $x(t) = (1 - u(t) - v(t))$ are unalterable constants; and (3), $\dot{\tilde{\lambda}}(t)/\tilde{\lambda}(t) = \dot{\tilde{\mu}}(t)/\tilde{\mu}(t)$, $\dot{\tilde{\eta}}(t)/\tilde{\eta}(t) = \dot{\tilde{\pi}}(t)/\tilde{\pi}(t)$. From the functional form of the utility function $U(c, x) = (c\varphi(x))^{1-\sigma}/(1-\sigma)$ and the definitions of $m(t)$ and $W(c(t), x(t), \Phi)$, we know that on the BGP,

$$\frac{\dot{U}_c(t)}{U_c(t)} = \frac{\dot{U}_{cx}(t)}{U_{cx}(t)} = \frac{\dot{W}_c(t)}{W_c(t)} = -\sigma\phi, \quad (21)$$

$$\frac{\dot{U}_x(t)}{U_x(t)} = \frac{\dot{U}_{xx}(t)}{U_{xx}(t)} = \frac{\dot{W}_x(t)}{W_x(t)} = \frac{\dot{m}(t)}{m(t)} = (1-\sigma)\phi. \quad (22)$$

Combining equations (14) and (22) gives rise to

$$\rho - \lambda - uG'(v) = (1-\sigma)\phi. \quad (23)$$

Taking the logarithmic derivatives with respect to t on both sides of equations (15)-(17) and substituting equations (18)-(22) into them, we know that on the BGP

$$-(\rho - \lambda) - \sigma\phi = \frac{\left[\left(\frac{\dot{\tilde{\mu}}(t)}{\tilde{\mu}(t)} - \sigma\phi \right) U_{cx}(t) - (F_k - \lambda) \frac{\dot{\tilde{\eta}}(t)}{\tilde{\mu}(t)} \right]}{U_{cx}(t) + \frac{\dot{\tilde{\eta}}(t)}{\tilde{\mu}(t)}}, \quad (24)$$

$$-(\rho - \lambda) + (1-\sigma)\phi = \frac{G'(v)^2 m(t) + \left(\frac{\dot{\tilde{\mu}}(t)}{\tilde{\mu}(t)} + (1-\sigma)\phi \right) U_{xx}(t) + [\phi - (F_k - \lambda)] \frac{\dot{\tilde{\eta}}(t)}{\tilde{\mu}(t)} F_n h(t)}{-\frac{\dot{\tilde{\lambda}}(t)}{\tilde{\mu}(t)} G' m(t) + U_{xx}(t) + \frac{\dot{\tilde{\eta}}(t)}{\tilde{\mu}(t)} F_n h(t)}, \quad (25)$$

$$-(\rho-\lambda)+(1-\sigma)\phi = \frac{uG'(v)G''(v)m(t) + \left[\frac{\dot{\tilde{\mu}}(t)}{\tilde{\mu}(t)} + (1-\sigma)\phi\right][m(t)G''(v) + U_{xx}(t)] + \left[(\phi - G) - \frac{\tilde{\eta}(t)}{\tilde{\pi}(t)}F_n u\right]\frac{\tilde{\pi}(t)}{\tilde{\mu}(t)}G'(v)h(t)}{-\frac{\tilde{\lambda}(t)}{\tilde{\mu}(t)}uG''(v)m(t) + [m(t)G''(v) + U_{xx}(t)] + \frac{\tilde{\pi}(t)}{\tilde{\mu}(t)}G'(v)h(t)}.$$
(26)

The left-hand side of equation (24) is a constant, i.e., $-(\rho - \lambda) - \sigma\phi$, which implies that the right-hand side is also constant on the BGP. Suppose that the growth rate of $\tilde{\mu}(t)$ is a constant, i.e., $\sigma\phi - (F_k - \lambda)$, that is,

$$\frac{\dot{\tilde{\mu}}(t)}{\tilde{\mu}(t)} - \sigma\phi = -(F_k - \lambda),$$
(27)

then the right-hand side is constant and equation (24) turns out to

$$F_k = \rho + \sigma\phi.$$
(28)

Substituting equation (27) into equation (25) gives rise to

$$-(\rho - \lambda) + (1 - \sigma)\phi = \frac{G'(v)^2 m(t) + [\phi - (F_k - \lambda)] \left[U_{xx}(t) + \frac{\tilde{\eta}(t)}{\tilde{\mu}(t)} F_n h(t) \right]}{-\frac{\tilde{\lambda}(t)}{\tilde{\mu}(t)} G' m(t) + U_{xx}(t) + \frac{\tilde{\eta}(t)}{\tilde{\mu}(t)} F_n h(t)}.$$
(29)

Suppose that the ratio $\tilde{\lambda}(t)/\tilde{\mu}(t)$ satisfies

$$G'(v) = -\frac{\tilde{\lambda}(t)}{\tilde{\mu}(t)}[\phi - (F_k - \lambda)].$$
(30)

Then the right-hand side of (29) is also constant and equation (29) degenerates to equation (28). Substituting equations (27) and (30) into equation (26) leads to

$$-(\rho-\lambda)+(1-\sigma)\phi = \frac{[\phi - (F_k - \lambda)] \left\{ -\frac{\tilde{\lambda}(t)}{\tilde{\mu}(t)} u G''(v) m(t) + [m(t)G''(v) + U_{xx}(t)] \right\} + \left[(\phi - G) - \frac{\tilde{\eta}(t)}{\tilde{\pi}(t)} F_n u \right] \frac{\tilde{\pi}(t)}{\tilde{\mu}(t)} G'(v) h(t)}{-\frac{\tilde{\lambda}(t)}{\tilde{\mu}(t)} u G''(v) m(t) + [m(t)G''(v) + U_{xx}(t)] + \frac{\tilde{\pi}(t)}{\tilde{\mu}(t)} G'(v) h(t)}.$$
(31)

Suppose the ratio $\tilde{\gamma}(t)/\tilde{\theta}(t)$ satisfies

$$(\phi - G) - \frac{\tilde{\eta}(t)}{\tilde{\pi}(t)} F_n u = \phi - (F_k - \lambda),$$
(32)

then equation (31) also degenerates to equation (28). Combining equations (27), (28), and (30), we obtain that

$$\frac{\dot{\tilde{\lambda}}(t)}{\tilde{\lambda}(t)} = \frac{\dot{\tilde{\mu}}(t)}{\tilde{\mu}(t)} = -(\rho - \lambda).$$
(33)

Similarly, combining equations (19), (28), and (32), we have that

$$\frac{\dot{\tilde{\eta}}(t)}{\tilde{\eta}(t)} = \frac{\dot{\tilde{\pi}}(t)}{\tilde{\pi}(t)} = -(F_k - \lambda) = -(\rho - \lambda + \sigma\phi).$$
(34)

Suppose that the economy approaches a balanced growth path satisfying the above three conditions. In the next section, we will prove the existence of such a balanced growth path. Now we examine the optimal capital income tax on the BGP. We have solved the Ramsey problem and found the steady state equation (28). In reinvestigating the competitive equilibrium, we have derived the Euler equation (4). Taking logarithmic derivatives on both its sides and substituting (7) and (21) into it, we have that on the BGP,

$$(1 - \tau) F_k = \rho + \sigma\phi.$$
(35)

Combining (28) and (35), we know that

$$\tau = 0,$$

which shows that the limiting capital income tax is zero, just as Lucas (1990) had conjectured.

4 Multiple equilibria: the existence of balanced growth paths

We examine the existence of the balanced growth path in this section. For this purpose, we let $z(t) = k(t)/u(t)h(t)$ to denote capital per unit of effective labor and $f(z) \equiv F(z, 1) = z^\beta$ to denote production per unit of effective labor. Then a balanced growth path is described by the values of c/h , g/h , b/h , z , u , v and ϕ that satisfy:

$$u[f(z) - (\lambda + \phi)z] = \frac{c}{h} + \frac{g}{h}, \quad (36)$$

$$G(v) = \phi, \quad (37)$$

$$\rho + \sigma\phi = (1 - \tau)f'(z), \quad (38)$$

$$\frac{c}{h} \frac{\varphi'(x)}{\varphi(x)} = (1 - \theta)[f(z) - zf'(z)], \quad (39)$$

$$\rho - \lambda + (\sigma - 1)\phi = uG'(v), \quad (40)$$

$$\frac{g}{h} + \frac{b}{h} = \theta u[f(z) - zf'(z)] + z\tau u f'(z). \quad (41)$$

These equations are the BGP version of the technology description (3) and (9), the marginal conditions (4)-(7) and the government budget constraint (8). As is shown in Lucas (1990), we must treat one of the four fiscal variables (τ, θ, g, b) as endogenous, given the values of the other three. For simplicity, we set government transfers b to be zero. As is shown in the Ramsey problem, we derive the limiting capital tax as zero ($\tau = 0$). If taking θ as given, then we only need to pin down the endogenous value of g/h on the BGP. Plugging (41) into (36) to delete g/h , substituting (36) into (39) to delete c/h , and putting the assumed functional forms of φ , G , and F , we obtain the equation system about (u, v, z) as follows:

$$u[z^\beta - (\lambda + Dv^\gamma)z] = \left[\frac{(1 - \theta)(1 - u - v)}{\alpha} + \theta u \right] (1 - \beta) z^\beta, \quad (42)$$

$$\rho + \sigma Dv^\gamma = \beta z^{\beta-1}, \quad (43)$$

$$\rho - \lambda + (\sigma - 1) Dv^\gamma = u D\gamma v^{\gamma-1}. \quad (44)$$

Solving for z and u from equations (43) and (44) respectively, and substituting them into equation (42), we have that

$$\frac{[\rho - \lambda + (\sigma - 1) Dv^\gamma][\rho + \sigma Dv^\gamma - \beta(\lambda + Dv^\gamma)]}{D\gamma v^{\gamma-1}(\rho + \sigma Dv^\gamma)(1 - \beta)} = \frac{[\alpha\theta - (1 - \theta)][\rho - \lambda + (\sigma - 1) Dv^\gamma] + (1 - \theta)(1 - v) D\gamma v^{\gamma-1}}{\alpha D\gamma v^{\gamma-1}},$$

which is equivalent to

$$\left(\frac{\{\alpha - (1 - \beta)[(1 + \alpha)\theta - 1]\}(\rho + \sigma Dv^\gamma)}{-\alpha\beta(\lambda + Dv^\gamma)} \right) [\rho - \lambda + (\sigma - 1) Dv^\gamma] = (1 - \beta)(1 - \theta)(1 - v) D\gamma v^{\gamma-1}(\rho + \sigma Dv^\gamma).$$

This is a very complex nonlinear algebraic equation about v . With the help of appropriate guesses, we display the possibility of multiple equilibria (i.e., multiple BGPs). Even though there may exist many BGPs, we just list three ones as follows:

BGP 1. Let $\rho - \lambda + (\sigma - 1) Dv^\gamma = \sigma Dv^\gamma$. We solve it for $v = [(\rho - \lambda)/D]^{1/\gamma}$. Substituting v into (43) and (44), we have one BGP satisfying

$$(v_1, u_1, z_1) = \left(\left(\frac{\rho - \lambda}{D} \right)^{1/\gamma}, \frac{\sigma}{\gamma} \left(\frac{\rho - \lambda}{D} \right)^{1/\gamma}, \left(\frac{\rho + \sigma(\rho - \lambda)}{\beta} \right)^{1/(\beta-1)} \right),$$

with a balanced growth rate $\phi_1 = \rho - \lambda (> 0)$.

BGP 2. Let $\rho - \lambda + (\sigma - 1) Dv^\gamma = (2\sigma - 1) Dv^\gamma$. By the same procedure, we solve another BGP:

$$(v_2, u_2, z_2) = \left(\left(\frac{\rho - \lambda}{\sigma D} \right)^{1/\gamma}, \frac{2\sigma - 1}{\gamma} \left(\frac{\rho - \lambda}{D\sigma} \right)^{1/\gamma}, \left(\frac{2\rho - \lambda}{\beta} \right)^{1/(\beta-1)} \right),$$

with a balanced growth rate $\phi_2 = (\rho - \lambda) / \sigma$.

BGP 3. Let $\rho - \lambda + (\sigma - 1) Dv^\gamma = (3\sigma - 1) Dv^\gamma$. Then we solve the third BGP:

$$(v_3, u_3, z_3) = \left(\left(\frac{\rho - \lambda}{2\sigma D} \right)^{1/\gamma}, \frac{3\sigma - 1}{\gamma} \left(\frac{\rho - \lambda}{2\sigma D} \right)^{1/\gamma}, \left(\frac{3\rho - \lambda}{2\beta} \right)^{1/(\beta-1)} \right),$$

with a balanced growth rate $\phi_3 = (\rho - \lambda) / 2\sigma$.

5 Conclusion

In this paper, by introducing a new state variable and changing the integral equation constraint into a differential equation and an identity, we solve the Ramsey problem with endogenous learning brought forward by Lucas (1990) and prove Lucas (1990)'s conjecture on zero limiting capital tax. Furthermore, we solve the balanced growth paths explicitly and show the possibility of multiple equilibria with zero capital tax.

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